A Scheme to Measure Error Rates in the Presence of Adjacent Channel Interference using Frequency Domain Analysis

Brijraj Vaghani
School of Computing and Engineering
University of Missouri Kansas City
Email: bbvkb8@umkc.edu

Cory Beard
School of Computing and Engineering
University of Missouri Kansas City
Email: beardc@umkc.edu

***Abstract*— This paper presents a scheme to measure the error rate of the communication system in the presence of Adjacent Channel Interference using frequency domain signal processing. Also studied is the performance of M-PSK and M-QAM signals received in the presence of Adjacent Channel Interference. The model of the system considers interfering signals of the same modulation format placed symmetrically around the main channel, rectangular pulse shaping, randomized phase and amplitude of the transmitted signals and timing misalignment between the channels. Expression for bit error rate corresponding to the approximate probability density function of the received signal is obtained. Curves of bit error rate as a function of order of modulation and distance between the channels are presented.**

***Index Terms*—Autocorrelation, Power Spectral Density, Probability Distribution function, Threshold.**

I. Introduction

All throughout the evolution of mobile communication, the design philosophy of the radio system has been influenced by the ever increasing demand of the available limited spectrum for voice and data applications. GSM technology has been up to this challenge by evolving from the first voice only networks to the incorporation of Short Message Service (SMS) followed by High Speed Circuit Switched Data (HSCSD) followed by the General Packet Radio Service (GPRS). After GPRS, further improvement in data rates is being achieved with the Enhanced Data rate for next Generation Evolution (EDGE) technology in which higher data rates are obtained by transmitting more bits per symbol which is achieved by migrating from the use of non linear modulation techniques (eg: GMSK) to higher order linear modulation techniques (eg: M-PSK) and with the use of multicarrier systems. This migration results in an increase in the bandwidth efficiency but at the same time has drawbacks of poor power efficiency, increased intersymbol and adjacent channel interference.

In this paper we present a scheme to measure the error rate of a system degraded due to the effects of Adjacent Channel Interference using the power spectral density of the main channel and the interfering channel. This approach is then used to find out the degradation due to adjacent channel interference in M-PSK systems and M-QAM systems, which form the basis of EDGE technology. Earlier work has been done to find out the error rate caused due to adjacent channel interference for MSK systems [3], QPSK, 8-PSK, 16-QASK and QPR systems [4], but the equations derived for the above systems are not scalable when the order of modulation is of the order of 32 or more. Error probability equations for a multilevel continuous phase shift keyed systems in the presence of intersymbol, interchannel and co-channel interference are derived in [5], in which expressions for binary CPSK are derived whereas upper bound and lower bounds are presented for multilevel systems. Error rate considerations for 256-QAM systems in the presence of Adjacent Channel Interference are given in [1], with the help of a simulation model, but it does not provide a closed form expression of the bit error rate.

In this paper we use a generalized approach to find out the amount of interference and also its effect, and the approach is applicable irrespective of the type and order of modulation and number of interfering channels. The expression for probability of error is obtained without any prior assumption about the stochastic processes involved. Using a statistical approach mentioned in [2], the features of a decision variable are extracted and matched to the statistical features of other models and the model with the closest match is selected. Each model has an associated error probability based on a threshold beyond which errrors occur; this can be used as an estimation of system error probability.

II. Model of the System

Consider the model shown in Fig. 1, for discussion assume that the modulator is M-PSK. As shown, it consists of three different channels 1, 2 and 3 operating at frequencies f_{c1}, f_{c2}, and f_{c3} respectively, where channel 1 is the main channel and the other two are the adjacent interfering channels placed symmetrically around the main channel. The model consists of three random bit generators which generate bit sequences b_{k1}, b_{k2} and b_{k3} with bits equally likely. These bit sequences are passed through a symbol former which forms symbols A_{k1}, A_{k2} and A_{k3} respectively depending on the order of the modulation. These symbols serve as an input to the modulators, at the output of which we get the following:

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} cos(2\pi f_{c1} t + \theta_1) \tag{1a}$$

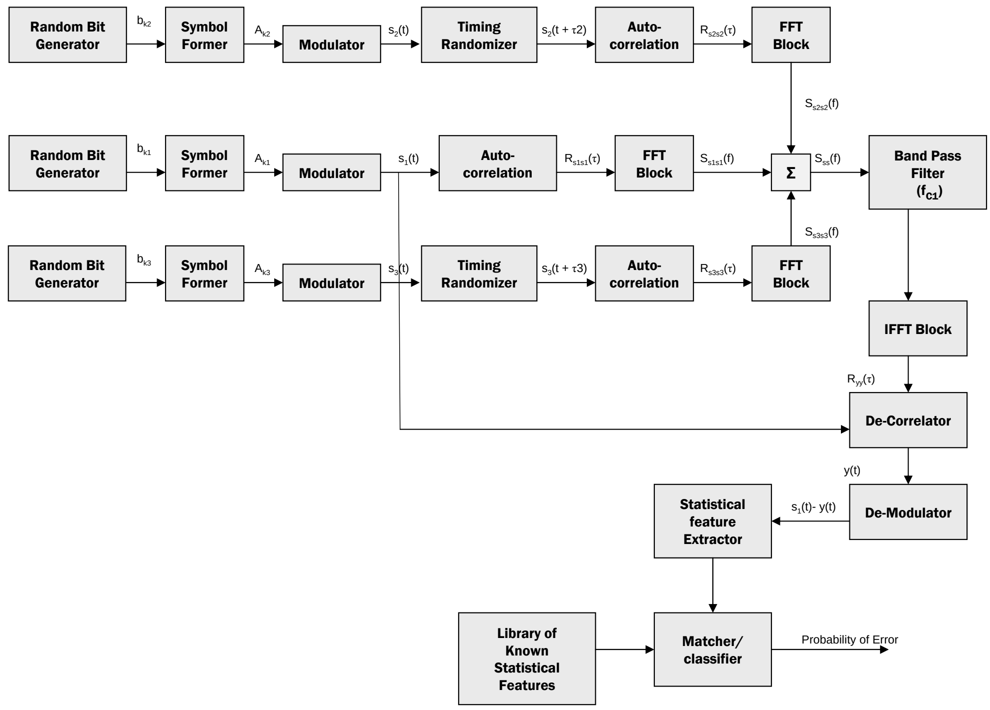

Fig. 1. Overall System Block Diagram

$$s_2(t) = \sqrt{\frac{2E_s}{T_s}} cos(2\pi f_{c2} t + \theta_2) \tag{1b}$$

$$s_3(t) = \sqrt{\frac{2E_s}{T_s}} cos(2\pi f_{c3} t + \theta_3) \tag{1c}$$

where E_s is the energy per symbol and T_s is the time duration of each symbol, and s_1(t), s_2(t) and s_3(t) are random processes, that are functions of the random phase angles θ_1, θ_2 and θ_3 respectively. The values of these phase angles are given by

$$\theta = \frac{A_{k1} 2\pi}{M} \tag{2}$$

The signals in the two adjacent channels are passed through a timing randomizer to introduce a time shift in the interfering signal with respect to the main signal. The randomizer introduces a time shift of τ_2 and τ_3 in channels 2 and 3 respectively, where τ_2 and τ_3 are discrete random variables that can take equiprobable values between 0 and $T/2$; at the output of the randomizer we get

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} cos(2\pi f_{c1} t + \theta_1) \tag{3a}$$

$$s_2(t) = \sqrt{\frac{2E_s}{T_s}} cos(2\pi f_{c2} (t + \tau_2) + \theta_2) \tag{3b}$$

$$s_3(t) = \sqrt{\frac{2E_s}{T_s}} cos(2\pi f_{c3} (t + \tau_3) + \theta_3) \tag{3c}$$

For the sake of the decorrelation operation described later, each signal is concatenated with a flipped version of itself (reversed in time) so that the result is symmetric in time about its center point. For further discussion we assume that the symmetric signal is represented in the same way as its original signal. The next step is to perform the frequency domain analysis for which we need to find the Power Spectral Density of the random process. To do so, the first step is to find the autocorrelation function of the random process given in (3)

$$R_{s1s1}(\tau) = E[s_1(t) s_1(t + \tau)] \tag{4a}$$

$$R_{s2s2}(\tau) = E[s_2(t) s_2(t + \tau)] \tag{4b}$$

$$R_{s3s3}(\tau) = E[s_3(t) s_3(t + \tau)] \tag{4c}$$

The autocorrelation can also be formed by convolving the signal with its time reversed version if we assume all time-shift combinations are equally likely. The Power Spectral Density of a random process is the Fourier Transform of its autocorrelation function. Taking the Fourier Transform of (4)

$$S_{s1s1}(f) = \int_{-\infty}^{\infty} R_{s1s1}(\tau) e^{-j2\pi f \tau} \; df \tag{5a}$$

$$S_{s2s2}(f) = \int_{-\infty}^{\infty} R_{s2s2}(\tau) e^{-j2\pi f \tau} \; df \tag{5b}$$

$$S_{s3s3}(f) = \int_{-\infty}^{\infty} R_{s3s3}(\tau) e^{-j2\pi f \tau} \; df \tag{5c}$$

The above calculation mentioned in (5) can be approximated using the Fast Fourier Transform. The functions in (5) are passed through an adder producing a resulting power spectral density function given by

$$S_{ss}(f) = S_{s1s1}(f) + S_{s2s2}(f) + S_{s3s3}(f) \tag{6}$$

This result of (5) is passed through a bandpass filter with cut-off frequency centered around f_{c1} using transfer function $H(f)$. The output of the bandpass filter is:

$$S_{yy}(f) = S_{ss}(f)H(f) \tag{7}$$

Now S_{yy} is the power spectral density of the received signal that has been affected by adjacent channel interference. At the receiving end, the time domain representation of the signal (7) is obtained by taking the Inverse Fourier Transform given by,

$$R_{yy}(\tau) = \int_{-\infty}^{\infty} S_{yy}(\tau)e^{-j2\pi f\tau}\ df \tag{8}$$

Equation (8) represents the autocorrelation function of the interference-affected signal. Finally, the interference-affected signal can be obtained by reversing the effect of the autocorrelation function. This reversing without the presence of interference or noise can be done by directly deconvolving the time reversed version of $s_1(t)$ from $R_{yy}(\tau)$, but in the presence of noise or interference this causes undesired noise spikes in the received signal and changes its statistical properties. To eliminate the effect of these noise spikes, the following procedure can be followed for deconvolution.

For simplicity, we represent a sampled version of a continuous signal in polynomial form, where sampled values are coefficients of the polynomial. Let $R_{yy}(x)$ be the representation for $R_{yy}(\tau)$, $s(x)$ be that for $s_1(t)$, $s_r(x)$ be the representation for the time reversed version of $s_1(t)$, $i(x)$ represent the interference signal and $i_r(x)$ the time reversed version of the interference signal.

Autocorrelation in polynomial form can be represented by the multiplication of the polynomial with its flipped version.

$$R_{ss}(x) = s(x)s_r(x) \tag{9}$$

Since $y(x) = s(x) + i(x)$, from (9) the result is

$$R_{yy}(x) = (s(x) + i(x))(s_r(x) + i_r(x)) \tag{10a}$$

$$R_{yy}(x) = s(x)s_r(x) + i(x)s_r(x) + i_r(x)s(x) + i(x)i_r(x) \tag{10b}$$

By the way $s_1(t)$ was formed, $s(x) = s_r(x)$. We also assume $i(x) = i_r(x)$ and $i(x)i_r(x)$ can be assumed to go to 0 (the interference has a much lower magnitude as compared to the desired signal). This results in

$$\frac{R_{yy}(x)}{s_r(x)} = s(x) + i(x) + i_r(x) \tag{11a}$$

$$\frac{R_{yy}(x)}{s_r(x)} = s(x) + 2i(x) \tag{11b}$$

The received deconvolved signal will be the sum of the original signal and interference.

$$y(x) = s(x) + i(x) = 0.5(\frac{R_{yy}}{s_r(x)} + s(x)) \tag{12}$$

Hence the deconvolution can be done by dividing the output of (8) by the time reversed version $s_1(t)$, adding it to $s_1(t)$ and then scaling the sum by 0.5.

The signal obtained from (12) is then passed through the demodulator (M-PSK for the purpose of discussion) to obtain the in-phase and quadrature channel signals as

$$r_{1k} = \int_{nT}^{(n+1)T} y(t)\sqrt{\frac{2E_s}{T_s}}cos(2\pi f_{c1}t)\ dt \tag{13a}$$

$$r_{2k} = -\int_{nT}^{(n+1)T} y(t)\sqrt{\frac{2E_s}{T_s}}sin(2\pi f_{c1}t)\ dt \tag{13b}$$

The phase of the received signal is given by

$$\theta_r = tan^{-1}\frac{r_{2k}}{r_{1k}} \tag{14}$$

Then to find the probability of error, first the deviation of the received values from the corresponding transmitted values is calculated

$$deviation = \theta - \theta_r \tag{15}$$

The probability of error is found from the deviation given in (15), here the characteristics of the random variable in (15) governs the occurrence of errors. As shown, the output of the demodulator goes to the feature extractor which extracts features of the random variable in (15), then the classifier performs a matching process, which involves choosing a model whose statistical features most closely resemble the features of the variable in (15). Matching is performed using the Chi-Square goodness of fit,

$$\chi^2 = \sum_{i=1}^{M} \frac{(\mu_j - e_j)^2}{e_j} \tag{16}$$

where M is the number of bins of a histogram, μ_j is the count in a particular bin, and e_j is the expected value of the bin in the distribution function. A small value of χ^2 indicates the two distributions are alike, and hence we can choose that distribution. On performing the test mentioned in (16) on the variable in (15) it was found that value of χ^2 is the lowest for the Exponential distribution in the case of an M-PSK system and the lowest for the Gamma distribution for an M-QAM system.

The general equation for probability of error is given by :

$$P_e = 1 - \int_0^T f_x(x)dx \tag{17}$$

where $f_x(x)$ is the chosen probability density function. The exponential probability density function for non-negative x is given by

$$f_x(x) = \lambda e^{-\lambda x} \tag{18}$$

where parameter λ is positive. The gamma probability density function for nonnegative x is given by

$$f_x(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta} \tag{19}$$

where α and β are greater than 0 and $\Gamma()$ is defined as the gamma function defined as

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y}\, dy \tag{20}$$

and T is the threshold. For M-PSK and M-QAM systems, thresholds are given as

$$T_{M-PSK} = \frac{2\pi}{2 * no.of bits/symbol} \tag{21a}$$

$$T_{M-QAM} = 0.5 * (Distance between constellation points) \tag{21b}$$

III. Results and discussion

In this section we present three sets of results:

The first set of results shows how close the approximated distribution is to the histogram of the received random process. Fig. 2 and Fig. 3 show a bar chart representation for the histogram of the deviation caused in the received signal due to Adjacent Channel Interference for a 16-PSK system and 16-QAM system respectively. Imposed on the bar chart in Fig. 2 is the exponential and on Fig. 3 is the gamma distribution. The results show that the histograms closely approximate the distribution function.

The second and third sets of results are obtained using Monte-Carlo simulation. The second set shows the relationship between the orders of the modulation and the amount of degradation due to Adjacent Channel Interference with average power per symbol, bit timing, and frequency separation between the channels the same in all cases. Fig. 4 and Fig. 5 show the curve of Bit Error Rates versus the order of modulation for M-PSK and M-QAM systems respectively. It can be seen that the Bit Error Rate increases with the order of the modulation; also it can be seen that the Bit Error Rate for M-PSK systems is much lower than that for M-QAM systems at the same average power.

In the last set of results, we show the effect of frequency separation on the Adjacent Channel Interference for 64-QAM and 64-PSK with the average power per symbol and bit timing the same for both the cases. As can be seen in Fig. 6 and Fig. 7, the amount of degradation in Bit Error Rate decreases with increasing channel spacing.

IV. Conclusion

In this paper we studied a generalized method to study the degradation due to the presence of Adjacent Channel Interference. Comparisons of error rate performance of M-PSK systems and M-QAM systems for different parameters are provided. The results show that the amount of interference increases with the order of the modulation, decreases with increasing channel spacing and is relatively greater for M-QAM

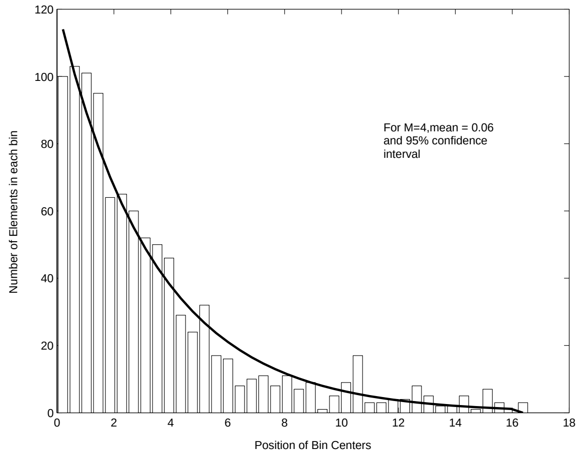

Fig. 2. Probability Density function for 16-PSK system

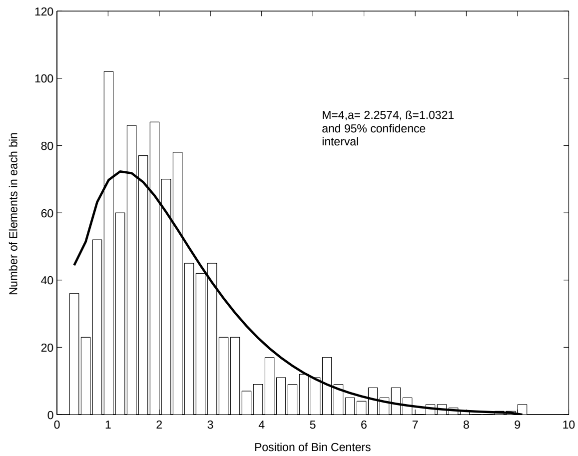

Fig. 3. Probability Density function for 16-QAM system

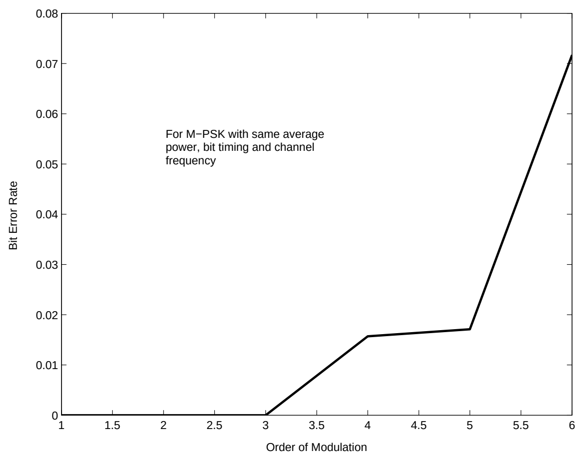

Fig. 4. Bit Error Rate as a function of order of modulation Rate for M-PSK system

systems than for M-PSK systems. All the results conform with the existing theory and hence support the validity of the scheme. Now this approach can readily be used to examine higher order modulation schemes and comparisons between modulation schemes without becoming overly concerned that the order or type of modulation might make analysis intractable.

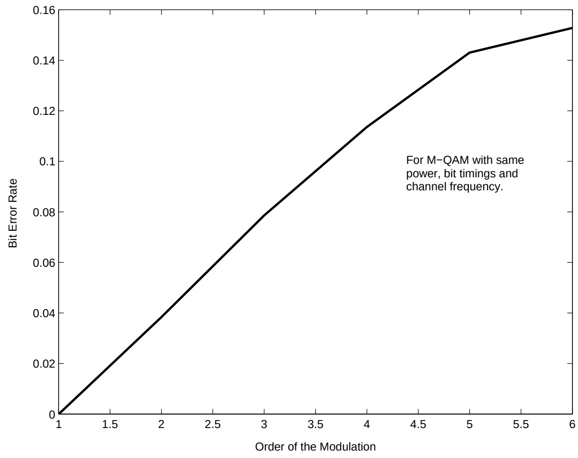

Fig. 5. Bit Error Rate as a function of order of modulation for M-QAM system

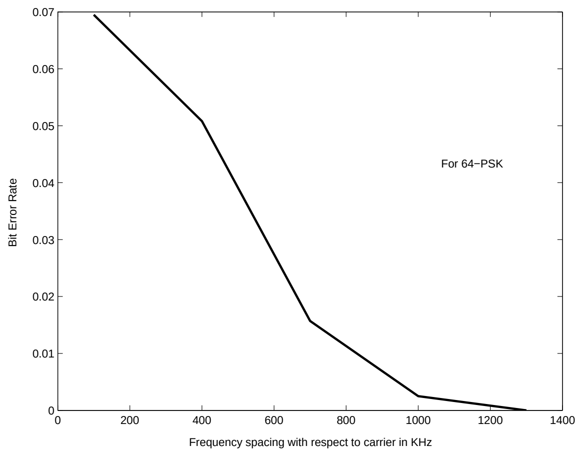

Fig. 6. Bit Error Rate as a function of frequency for 64-PSK system

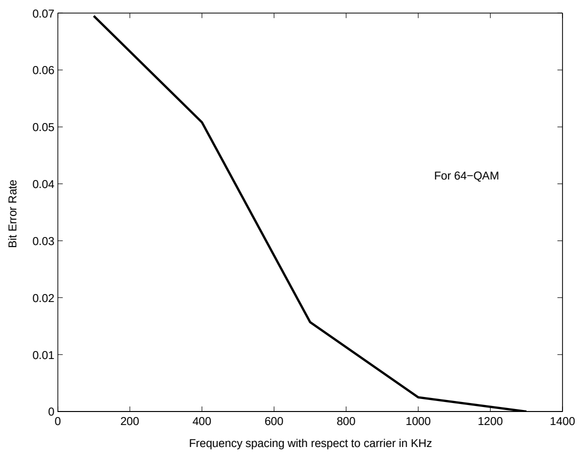

Fig. 7. Bit Error Rate as a function of frequency for 64-PSK system

REFERENCES

[1] Panayotis Mathiopoulous, Kamilo Feher, "Error Rate Consideration for 256-QAM Systems in the Presence of Adjacent Channel Interference," *IEEE*, pp. 1064-1068, 1988.

[2] Jason B. Scholz, Bruce R. Davis, "Error Probability Estimator Structure Based Analysis of the Receiver Design Variable," *IEEE transactions on communications* , vol. 43, no.8,, pp. 2311-2315 August 1995.

[3] Israel Korn, Bernie Seth, "Adjacent-Channel and Quadrature-Channel Interference in Minimum Shift Keying ," *IEEE journal on selected areas of communications*, vol.sac-1, pp. 21-28, January 1983.

[4] Laurence B. Milstein, Raymond L. Pickholtz, Donald L. Schilling, "Comparison of Performance of Digital Modulation Techniques in the presence of Adjacent Channel Interference," *IEEE transactions on Communications*, vol.com-30, pp. 1984-1993, August 1982.

[5] Sergio Benedetto, Ezio Biglieri, Valentino Castellani, "Combined Effects of Intersymbol, Interchannel, and Co-Channel Interferences in M-ary CPSK systems," *IEEE transactions on communication*,vol.com-21,no.9, pp. 997-1008, September 1973.

[6] M.Jeruchim, P. Balaban, K.S.Shanmugan *Simulation of Communication System*, Plenum Press, 1992.